Quiz 6, Linear

Name: Key

1. (4 points) Let W be the set of all vectors of the form $\begin{bmatrix} 2a-3b \\ -1 \\ 2a-5b \end{bmatrix}$, where a, b are arbitrary real numbers. Either find a set S that spans W or give an example (or explanation) to show that W is *not* a vector space.

W is not a vector space b/c $\vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is not in W, since second entry is always -1.

2. (3 points) Suppose $\mathbb{R}^4 = \text{Span}\,\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. *Use the definition of basis*, as well as the Invertible Matrix Theorem, to explain why $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$ is a basis for $\mathbb{R}^4$.

v_2, v_3

To be a basis, needs to (1) span $\mathbb{R}^4$ and (2) be linearly indep. Clearly $v_1, \ldots, v_4$ span $\mathbb{R}^4$ (given). Each vector must have 4 entries b/c they are in $\mathbb{R}^4$, so they would form a square matrix $[v_1\ v_2\ v_3\ v_4]$. By I.M.T, if they span then the matrix is invertible, so the columns are linearly independent $\Rightarrow$ it is a basis since it has both (1) and (2).

3. (3 points) Let $M_{2\times 2}$ be the vector space of all 2×2 matrices, and define $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that T is a linear transformation.

$T(A+B) = (A+B) + (A+B)^T = A+B+A^T+B^T = (A+A^T)+(B+B^T)$
$= T(A) + T(B)$ ✓

$T(cA) = cA + (cA)^T = cA + cA^T = c(A+A^T) = cT(A)$ ✓

5 min

4. (1 point, Extra Credit) Let $A = \begin{bmatrix} 2 & 3 & -2 & 0 & 4 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$. Find a nonzero vector in Nul A and a nonzero vector in Col A. Explain why your answers are correct.

- $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ is in Nul A b/c $\begin{bmatrix} 2 & 3 & -2 & 0 & 4 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

- Any column is in Col A. So $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is in Col A.